

Polyakov loop and QCD thermodynamics from the gluon and ghost propagators

Kenji Fukushima

Department of Physics, Keio University, Kanagawa 223-8522, Japan

Kouji Kashiwa

RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

We investigate quark deconfinement by calculating the effective potential of the Polyakov loop using the non-perturbative propagators in the Landau gauge measured in the finite-temperature lattice simulation. With the leading term in the 2-particle-irreducible formalism the resultant effective potential exhibits a first-order phase transitions for the pure SU(3) Yang-Mills theory at the critical temperature consistent with the empirical value. We also estimate the thermodynamic quantities to confirm qualitative agreement with the lattice data near the critical temperature. We then apply our effective potential to the chiral model study and calculate the order parameters and the thermodynamic quantities. Unlike the case in the pure Yang-Mills theory the thermodynamic quantities are sensitive to the temperature dependence of the non-perturbative propagators, while the behavior of the order parameters is less sensitive, which implies the importance of the precise determination of the temperature-dependent propagators.

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Introduction It has been a long-standing question in physics of quantum chromodynamics (QCD) how to understand confinement of quarks and gluons in the vacuum and how to clarify the nature of deconfinement in a medium in extreme environments (see Ref. [1] for reviews).

It was Polyakov [2] who first addressed the deconfinement phase transition successfully in the strong-coupling limit of a pure Yang-Mills theory. The order parameter for deconfinement was then identified, which is now called the Polyakov loop. Later on, the strong-coupling expansion was extended to implement quarks and the chiral dynamics [3]. One of the most popular approaches in the QCD phase-diagram research, i.e. the chiral effective model such as the Nambu–Jona-Lasinio model [4–7], the linear-sigma model [8], etc [9] with the Polyakov loop, is a natural extension from the strong-coupling QCD.

The largest ambiguity in the P-chiral models lies in the choice of the effective potential of the Polyakov loop. Though the initial choice was motivated by the strong-coupling expansion [4], it is now common to adopt the potential that fits the pure Yang-Mills thermodynamics from the lattice simulation either in a polynomial form [5] or in a Haar-measure form [4, 6]. Since the fitting procedure does not refer to microscopic dynamics at all, it is unclear how the Polyakov-loop potential is related or unrelated to non-perturbative characteristics near T_c as studied in the matrix model [10] (see also Ref. [11] for an insight into the physical contents of the potential).

An important breakthrough came from an attempt to understand quark deconfinement in terms of the Landau-gauge propagators that describe gluon confinement [12, 13]. In the Landau gauge it is the deep-infrared enhancement in the ghost propagator that causes confinement, while the gluon propagator is infrared suppressed. This behavior is qualitatively consistent with the confinement scenarios by Kugo and Ojima (they are equivalent

if the ghost renormalization factor diverges at zero momentum [14]) and also by Gribov and Zwanziger [15]. Indeed the gluon and ghost propagators in the Landau gauge at zero and finite temperature have been calculated in the lattice simulation [16–18], the Dyson-Schwinger equation (DSE) [19, 20], the functional renormalization group (FRG) approach [13, 20, 21].

In this work we report on an extension of Ref. [13] using the T -dependent propagators from the lattice-QCD simulation [18] (see Ref. [22] for the finite- T propagators from the FRG calculation). Also, we try to address the thermodynamic quantities constructed from the propagators. We would stress here that our central point is not only an extension of Ref. [13] but it should aim to build a bridge over the model studies and the first-principle functional approaches. Such a work must be extremely useful for both sides; there are many arguments to suggest that the back-reaction from the matter to the glue sector has crucial impacts on the QCD phase-diagram research based on the P-chiral models [8, 23]. In principle this shortcoming of insufficient back-reaction in the P-chiral models could be resolved with the DSE or FRG approaches [24], but then one has to confront huge-scale computations. As we see later, thanks to a nice parametrization of the propagators in Ref. [18], our practical strategy requires only minimal modifications in the model-study procedures. The outcomes are quite promising as we will see.

Propagator and Parametrization In the covariant gauge the gluon propagator inverse can be decomposed into the transverse and the longitudinal parts using the projection operators;

$$T_{\mu\nu} \equiv g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad L_{\mu\nu} \equiv g_{\mu\nu} - T_{\mu\nu}, \quad (1)$$

T	r_t^2 (GeV ²)	d_t (GeV ⁻²)	r_l^2 (GeV ²)	d_l (GeV ⁻²)
$0.86T_c$	0.880	0.143	0.256	0.220
$1.20T_c$	0.963	0.140	1.018	0.162

TABLE I. Parameters for the non-perturbative propagators at $T = 0.86T_c$ and $1.20T_c$ taken from Ref. [18].

and the further decomposition relative to the rest frame at finite temperature,

$$P_{\mu\nu}^T \equiv (1 - \delta_{\mu 4})(1 - \delta_{\nu 4}) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{\mathbf{p}^2} \right), \quad P_{\mu\nu}^L \equiv T_{\mu\nu} - P_{\mu\nu}^T. \quad (2)$$

The gluon propagator in the Landau gauge is then parametrized as

$$D_{A\mu\nu}(p^2) = \delta^{ab} \left(D_T^{(T)} P_{\mu\nu}^T + D_T^{(L)} P_{\mu\nu}^L + \xi D_L L_{\mu\nu} \right). \quad (3)$$

Interestingly these propagators as well as the ghost propagator D_C are compactly expressed in a Gribov-Stingl form;

$$\begin{aligned} D_T^{(T)} &\propto \frac{d_t(p^2 + d_t^{-1})}{(p^2 + r_t^2)^2}, & D_L &= \frac{1}{p^2}, \\ D_T^{(L)} &\propto \frac{d_l(p^2 + d_l^{-1})}{(p^2 + r_l^2)^2}, & D_C &= \frac{p^2 + d_g^{-1}}{(p^2)^2}, \end{aligned} \quad (4)$$

as discussed in Refs. [18, 25]. This form also appears in a low-energy effective description of the Yang-Mills theory [26]. We note that we postulated non-renormalization for the longitudinal gluon propagator because $Z_L = 1 + \mathcal{O}(\xi)$ and $\xi \rightarrow 0$ is taken after all in the Landau gauge. In this work we use two sets of parameters at $T = 0.86T_c$ and $T = 1.20T_c$ taken from Ref. [18], which is listed in Tab. I. The ghost propagator can be well reproduced by a choice of $d_g^{-1} = 0.454$ GeV² which is nearly independent of T .

Polyakov Loop Potential The Gribov-Stingl form is especially convenient for the practical computation of the partition function and the effective potential of the Polyakov loop. In the present study, we make use of an approximation motivated from the 2-particle-irreducible (2PI) formalism or the FRG equation. In the 2PI formalism for example, the full effective action can be expressed generally as

$$\Gamma = \frac{1}{2} \text{tr} \ln G^{-1} - \frac{1}{2} \text{tr} \ln (G^{-1} - G_0^{-1}) G + \Gamma_2[G] \quad (5)$$

with the full propagator G , the tree-level propagator G_0 , and the sum of the 2PI diagrams $\Gamma_2[G]$. Thus, all known approximations are derived from some truncation onto Eq. (5); the Hartree approximation for example picks up the first term $\text{tr} \ln G^{-1}$ only with an assumption that G is written with a mean-field mass. Usually the full propagator should be fixed self-consistently within the 2PI formalism. Now that we already know the non-perturbative

propagators, however, we can directly plug them into Eq. (5) to have better results. In this “hybrid” method, the dominant contribution to the gluonic pressure comes from

$$\begin{aligned} \beta \Omega_{\text{glue}} &\simeq -\frac{1}{2} \text{tr} \ln D_A^{-1} + \text{tr} \ln D_C^{-1} \\ &= -\frac{1}{2} \text{tr} \ln D_L^{-1} - \text{tr} \ln D_T^{(T)-1} - \frac{1}{2} \text{tr} \ln D_T^{(L)-1} + \text{tr} \ln D_C^{-1}. \end{aligned} \quad (6)$$

We note that this leading-order truncation corresponds to the so-called quasi-particle model of thermodynamics [27]. In other words this method presumably works in the deconfinement regime where gluons are the physical degrees of freedom, while the approximation may fail the regime of glueballs at low T . Importantly, in principle, one can systematically improve the approximation by evaluating the sub-leading terms and Γ_2 diagrammatically once D_A and D_C are known.

For the purpose of calculating the effective potential for the Polyakov loop, we keep a constant A_4 in the temporal covariant derivative; p^2 in the gluon and ghost propagators are replaced with $\tilde{p}^2 = (2\pi T n + g\beta A_4)^2 + \mathbf{p}^2$ in the color adjoint representation. We note that the Polyakov loop is defined as

$$\Phi \equiv \frac{1}{3} \text{tr} L_3 = \frac{1}{3} \text{tr} \mathcal{P} \exp \left(ig \int_0^\beta A_4 dx_4 \right), \quad (7)$$

where \mathcal{P} represents the time ordering and A_4 is a matrix in the color fundamental representation.

Then, because the propagators (4) are written as a combination of $(p^2 + m^2)^{-1}$, it is straightforward to carry out the summation over the Matsubara frequency in an analytical way, i.e. for the transverse gluons for example, we have

$$\begin{aligned} \text{tr} \ln D_T^{(T)-1} &= 2 \text{tr} \ln (\tilde{p}^2 + r_t^2) - \text{tr} \ln (\tilde{p}^2 + d_t^{-1}) \\ &= 2W_B(r_t^2) - W_B(d_t^{-1}), \end{aligned} \quad (8)$$

where we have defined,

$$W_B(m^2) \equiv -2V \int \frac{d^3 p}{(2\pi)^3} \text{tr} \ln \left(1 - L_8 e^{-\beta \sqrt{\mathbf{p}^2 + m^2}} \right) \quad (9)$$

with L_8 being the Polyakov loop matrix in the color adjoint representation; $(L_8)_{ab} = 2 \text{tr}(t_a L_3 t_b L_3^\dagger)$. We dropped the divergent zero-point energy that is independent of T and thus an irrelevant offset. One can take the trace in color space explicitly to find [11],

$$W_B(m^2) = -2V \int \frac{d^3 p}{(2\pi)^3} \ln \left(1 + \sum_{n=1}^8 C_n e^{-n\beta \sqrt{\mathbf{p}^2 + m^2}} \right), \quad (10)$$

where $C_8 = 1$, $C_1 = C_7 = 1 - 9\bar{\Phi}\Phi$, $C_2 = C_6 = 1 - 27\bar{\Phi}\Phi + 27(\bar{\Phi}^3 + \Phi^3)$, $C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81(\bar{\Phi}\Phi)^2$, $C_4 = 2[-1 + 9\bar{\Phi}\Phi - 27(\bar{\Phi}^3 + \Phi^3) + 81(\bar{\Phi}\Phi)^2]$.

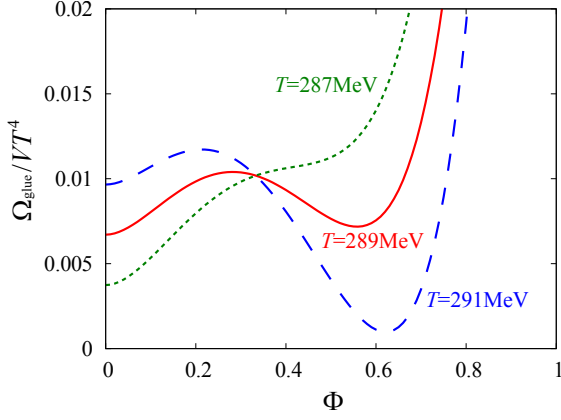


FIG. 1. Polyakov loop potential near T_c that exhibits a first-order phase transition as expected for the SU(3) case. With the parameter sets at $T = 0.86T_c$ and $T = 1.20T_c$ in Tab. I we find $T_c = 289$ MeV (as shown) and $T_c = 351$ MeV, respectively.

In this way, using Eqs. (6), (8), and (10), we can numerically calculate the Polyakov-loop effective potential $\beta\Omega_{\text{glue}}[\Phi]$, and then determine Φ from $\partial\Omega_{\text{glue}}[\Phi]/\partial\Phi = 0$. We find that the critical temperature in our treatment is $T_c = 289$ MeV and $T_c = 351$ MeV for the parameter sets at $T = 0.86T_c$ and $T = 1.20T_c$ in Tab. I, respectively. If we utilize the full T -dependent propagator, thus, the critical temperature should be somewhere in between. Considering the empirical value $T_c \sim 280$ MeV [28], one might wonder that our estimate of T_c is a bit too larger. This is fine because neglected Γ_2 has an effect to push T_c down [29]. This semi-quantitative agreement of T_c is amazing for its simplicity. The energy scales are provided through r^2 and d as given in Tab. I and there is no adjustable degrees of freedom. It is also worth mentioning that the Polyakov loop potential formulated here leads to a second-order phase transition for the color SU(2) case.

Although T_c is such different depending on the parameter sets, interestingly, Φ and all thermodynamic quantities as functions of T are *identical* if plotted in the unit of T_c . This is a quite non-trivial finding, and supports the validity of the fitted Polyakov loop potential characterized by only one dimensionful parameter T_0 [5, 6].

We show thermodynamic quantities as a function of T in Fig. 2 where the normalized pressure and the interaction measure are plotted. Because we adopt the parameter sets at $T = 0.86T_c$ and $T = 1.20T_c$ (yielding indistinguishable results if scaled with T_c), we choose the temperature range up to $1.3T_c$ here. We make a comparison with the lattice data and confirm semi-quantitative agreement. In particular, as seen in Fig. 2, the agreement of the pressure is pretty good, while the interaction measure does not fit the lattice data well.

The discrepancy above $\sim 1.2T_c$ should be attributed to the neglected T -dependence in the non-perturbative propagators as seen in Refs. [18, 31]. Once it is correctly taken into account, we expect that the overshoot of

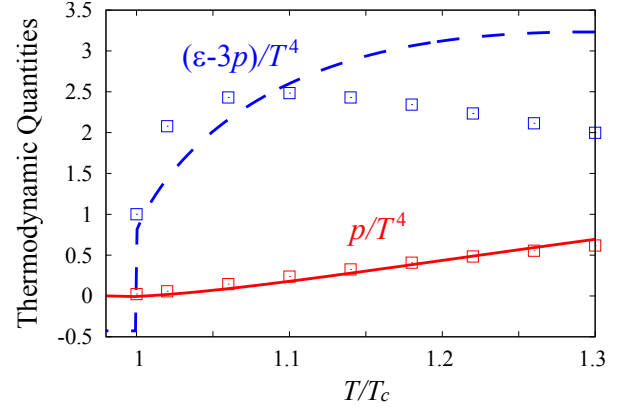


FIG. 2. Thermodynamic quantities as compared to the lattice results taken from a table in Ref. [30]. The numerical results are indistinguishable for the parameter sets of $T = 0.86T_c$ and $T = 1.2T_c$.

thermodynamic quantities could become milder. It should be mentioned that, in the $T \rightarrow \infty$ limit, thermodynamic quantities in our method approach the Stefan-Boltzmann limit, by construction, as all propagators go to $\sim 1/p^2$. Below T_c , on the other hand, we find that some of thermodynamic quantities go negative, which is caused by too strong ghost contributions. It is an unanswered question how to extract the expected behavior of the glueball gas [11, 30] from the gluon and the ghost propagators. One should cope with Γ_2 in Eq. (5) in the confinement regime. Although one may think that glueballs are too heavy to make a sizable contribution to thermodynamics, the electric glueballs can be significantly light in the vicinity of the (second-order) critical point [32].

Dynamical Quarks Just for the demonstration purpose of the usefulness of our effective potential let us apply it to an effective model. We emphasize that our goal is not to analyze the model itself but to seek for the possibility to utilize this *inverted Weiss potential* in phenomenology that is complementary to the field-theoretical argument [33]. To this end we adopt the covariant coupling in the (2+1)-flavor quasi-quark description. Then, the thermodynamic potential from the quark contribution reads,

$$\beta\Omega_{\text{quark}} = -6\beta V \sum_f \int^\Lambda \frac{d^3p}{(2\pi)^3} \sqrt{\mathbf{p}^2 + M_f^2} - 4 \sum_f W_F(M_f^2) + g_s (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2 + \langle \bar{s}s \rangle^2) + 4g_d \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle, \quad (11)$$

where $M_u = m_u - 2g_s \langle \bar{u}u \rangle - 2g_d \langle \bar{d}d \rangle \langle \bar{d}d \rangle$ etc and the last two terms above represent the condensation energy. Here we defined,

$$W_F(M^2) \equiv V \int \frac{d^3p}{(2\pi)^3} \text{tr} \ln \left(1 + L_3 e^{-\beta \sqrt{\mathbf{p}^2 + M^2}} \right), \quad (12)$$

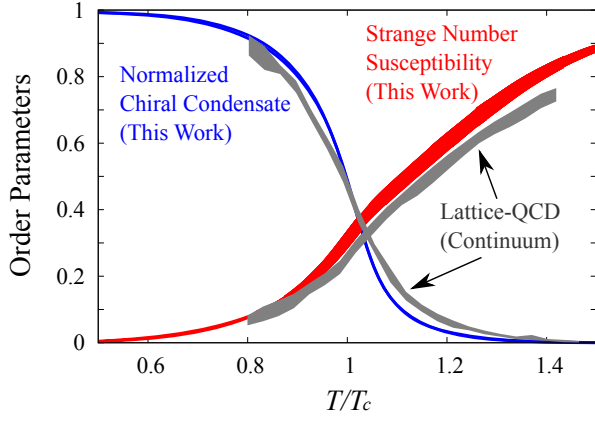


FIG. 3. Order parameters (see Eqs. (13) and (14) for definition) from our model calculation and the lattice-QCD results. The bands in our results correspond to the uncertainty from the parameter choices at $T = 0.86T_c$ (with $T_c = 182.5$ MeV for the unit of the horizontal axis) and $T = 1.20T_c$ (with $T_c = 191$ MeV). The gray bands are the lattice-QCD results in the continuum extrapolation from Refs. [35] (with $T_c = 156$ MeV).

in the color fundamental representation. Thus, we choose the parameters according to the standard set of the (2+1)-flavor NJL model as $\Lambda = 631.5$ MeV, $g_s\Lambda^2 = 3.67$, $g_d\Lambda^5 = -9.29$, $m_u = m_d = 5.5$ MeV, $m_s = 135.7$ MeV [34].

We can express $W_F(M^2)$ in terms of Φ [5] and solve the Polyakov loop Φ and the chiral condensates $\langle\bar{q}q\rangle$ to minimize the total potential $\Omega_{\text{glue}} + \Omega_{\text{quark}}$. Figure 3 shows our numerical results for the order parameters; The normalized chiral condensate is defined as

$$\Delta = \frac{\langle\bar{u}u\rangle - (m_{ud}/m_s)\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle_0 - (m_{ud}/m_s)\langle\bar{s}s\rangle_0}, \quad (13)$$

where $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$ represents the light-quark chiral condensate and the denominator is the value at $T = 0$. We adopt the strange-quark number susceptibility,

$$\chi_s/T^2 = \frac{1}{T^2} \frac{\partial n_s}{\partial \mu_s} = -\frac{1}{VT^2} \frac{\partial^2 \Omega}{\partial \mu_s^2} \quad (14)$$

for the deconfinement order parameter instead of the conventional Polyakov loop. This is because the Polyakov loop has a large renormalization factor, and in the mean-field approximation Φ should be considered as an internal parameter rather than an observable. To draw Fig. 3 we chose $T_c = 182.5$ MeV for the parameter set at $T = 0.86T_c$, $T_c = 191$ MeV for that at $T = 1.20T_c$, and the lattice-QCD results from Ref. [35] are plotted as a function of T/T_c with $T_c = 156$ MeV. One might have an impression that T_c in the model side is too large as compared to the lattice-QCD data, but this is to be improved with the back-reaction from the polarization diagrams [8, 23].

In this case with dynamical quarks thermodynamic quantities do not show unphysical behavior near T_c because quark degrees of freedom dominate over gluons.

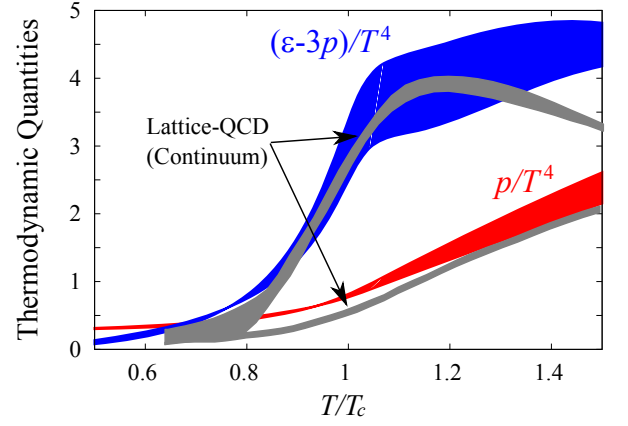


FIG. 4. Thermodynamic quantities as compared to the lattice-QCD results taken from Ref. [36]. The bands correspond to the uncertainty from the parameter choices at $T = 0.86T_c$ and $T = 1.20T_c$. The choice of T_c is the same as in Fig. 3.

The pressure and the interaction measure are plotted in Fig. 4. We see that our numerical results quantitatively agree with the lattice-QCD data taken from Ref. [36].

Our method with the momentum integration in Eq. (9) is simple enough to be an alternative of the Polyakov-loop potential used in the market of the P-chiral models. Moreover, it is advantageous for our method to be extendable to implement missed contributions such as the screening effects through the quark polarization diagrams. The improvement in this direction should be extremely important to tackle the realistic QCD phase structure especially at high baryon density or strong magnetic field [37] (for a recent review, see Ref. [38]). These effects do not directly couple to gluons, and nevertheless, the nature of deconfinement is affected substantially through the quark loops that carry the baryon number and the electric charge. Progresses in this direction shall be reported elsewhere.

Summary We elucidated how to construct the effective potential of the Polyakov loop from the non-perturbative propagators of gluon and ghost in the Landau gauge available from the lattice simulation. This is an extension of the idea of Ref. [13]. We took the fitting forms of the finite-temperature propagators from Ref. [18] and found a quite tractable way to calculate thermodynamic quantities as well as the order parameters as functions of T . We showed that the thermodynamic properties are nicely consistent with the lattice data in the vicinity of T_c . Furthermore we introduced dynamical quarks in the quasi-quark approximation to reproduce the simultaneous crossovers of deconfinement and chiral restoration. We made sure that our potential works well even on the quantitative level without fine-tuning of any parameter. It would be an intriguing future problem to apply our Polyakov-loop potential to the non-local version of the chiral model [39].

We believe that this present work takes one step forward to the understanding of the QCD phase diagram in extreme environments based on the first-principle-type calculations.

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- [1] K. Fukushima and T. Hatsuda, Rept. Prog. Phys. **74**, 014001 (2011), arXiv:1005.4814 [hep-ph]; K. Fukushima, J. Phys. G **G39**, 013101 (2012), arXiv:1108.2939 [hep-ph].
 - [2] A. M. Polyakov, Phys. Lett. **B72**, 477 (1978).
 - [3] E.-M. Ilgenfritz and J. Kripfganz, Z. Phys. **C29**, 79 (1985); A. Gocksch and M. Ogilvie, Phys. Rev. **D31**, 877 (1985).
 - [4] K. Fukushima, Phys. Lett. **B591**, 277 (2004), arXiv:hep-ph/0310121 [hep-ph].
 - [5] C. Ratti, M. Thaler, and W. Weise, Phys. Rev. **D73**, 014019 (2006), arXiv:hep-ph/0506234 [hep-ph].
 - [6] S. Roessner, C. Ratti, and W. Weise, Phys. Rev. **D75**, 034007 (2007), arXiv:hep-ph/0609281 [hep-ph].
 - [7] W.-j. Fu, Z. Zhang, and Y.-x. Liu, Phys. Rev. **D77**, 014006 (2008), arXiv:0711.0154 [hep-ph]; M. Ciminale, R. Gatto, N. Ippolito, G. Nardulli, and M. Ruggieri, Phys. Rev. **D77**, 054023 (2008), arXiv:0711.3397 [hep-ph]; Y. Sakai, K. Kashiwa, H. Kouno, and M. Yahiro, Phys. Rev. **D77**, 051901 (2008), arXiv:0801.0034 [hep-ph]; K. Fukushima, Phys. Rev. **D77**, 114028 (2008), arXiv:0803.3318 [hep-ph].
 - [8] B.-J. Schaefer, J. M. Pawłowski, and J. Wambach, Phys. Rev. **D76**, 074023 (2007), arXiv:0704.3234 [hep-ph]; B.-J. Schaefer, M. Wagner, and J. Wambach, Phys. Rev. **D81**, 074013 (2010), arXiv:0910.5628 [hep-ph]; T. K. Herbst, J. M. Pawłowski, and B.-J. Schaefer, Phys. Lett. **B696**, 58 (2011), arXiv:1008.0081 [hep-ph]; B. Schaefer and M. Wagner, Phys. Rev. **D85**, 034027 (2012), arXiv:1111.6871 [hep-ph].
 - [9] E. Megias, E. Ruiz Arriola, and L. Salcedo, Phys. Rev. **D74**, 114014 (2006), arXiv:hep-ph/0607338 [hep-ph].
 - [10] A. Dumitru, Y. Guo, Y. Hidaka, C. P. K. Altes, and R. D. Pisarski, Phys. Rev. **D83**, 034022 (2011), arXiv:1011.3820 [hep-ph]; Phys. Rev. **D86**, 105017 (2012), arXiv:1205.0137 [hep-ph].
 - [11] C. Sasaki and K. Redlich, Phys. Rev. **D86**, 014007 (2012), arXiv:1204.4330 [hep-ph]; M. Ruggieri, P. Alba, P. Castorina, S. Plumari, C. Ratti, *et al.*, Phys. Rev. **D86**, 054007 (2012), arXiv:1204.5995 [hep-ph].
 - [12] J. M. Pawłowski, D. F. Litim, S. Nedelko, and L. von Smekal, Phys. Rev. Lett. **93**, 152002 (2004), arXiv:hep-th/0312324 [hep-th].
 - [13] J. Braun, H. Gies, and J. M. Pawłowski, Phys. Lett. **B684**, 262 (2010), arXiv:0708.2413 [hep-th].
 - [14] T. Kugo, (1995), arXiv:hep-th/9511033 [hep-th].
 - [15] D. Zwanziger, Nucl. Phys. **B412**, 657 (1994).
 - [16] F. D. Bonnet, P. O. Bowman, D. B. Leinweber, and A. G. Williams, Phys. Rev. **D62**, 051501 (2000), arXiv:hep-lat/0002020 [hep-lat]; J. Gattnar, K. Langfeld, and H. Reinhardt, Phys. Rev. Lett. **93**, 061601 (2004), arXiv:hep-lat/0403011 [hep-lat]; A. Cucchieri and T. Mendes, PoS **LAT2007**, 297 (2007), arXiv:0710.0412 [hep-lat]; I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, **LAT2007**, 290 (2007), arXiv:0710.1968 [hep-lat]; T. Iritani, H. Suganuma, and H. Iida, Phys. Rev. **D80**, 114505 (2009), arXiv:0908.1311 [hep-lat].
 - [17] A. Cucchieri, T. Mendes, O. Oliveira, and P. Silva, Phys. Rev. **D76**, 114507 (2007), arXiv:0705.3367 [hep-lat]; A. Sternbeck, L. von Smekal, D. Leinweber, and A. Williams, PoS **LAT2007**, 340 (2007), arXiv:0710.1982 [hep-lat].
 - [18] R. Aouane, V. Bornyakov, E. Ilgenfritz, V. Mitrjushkin, M. Muller-Preussker, *et al.*, Phys. Rev. **D85**, 034501 (2012), arXiv:1108.1735 [hep-lat].
 - [19] L. von Smekal, R. Alkofer, and A. Hauck, Phys. Rev. Lett. **79**, 3591 (1997), arXiv:hep-ph/9705242 [hep-ph]; L. von Smekal, A. Hauck, and R. Alkofer, Annals Phys. **267**, 1 (1998), arXiv:hep-ph/9707327 [hep-ph]; R. Alkofer and L. von Smekal, Phys. Rept. **353**, 281 (2001), arXiv:hep-ph/0007355 [hep-ph]; C. Fischer, R. Alkofer, and H. Reinhardt, Phys. Rev. **D65**, 094008 (2002), arXiv:hep-ph/0202195 [hep-ph]; C. Fischer and R. Alkofer, Phys. Lett. **B536**, 177 (2002), arXiv:hep-ph/0202202 [hep-ph]; C. S. Fischer and J. M. Pawłowski, Phys. Rev. **D75**, 025012 (2007), arXiv:hep-th/0609009 [hep-th]; D. Dudal, S. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. **D77**, 071501 (2008), arXiv:0711.4496 [hep-th]; A. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. **D78**, 025010 (2008), arXiv:0802.1870 [hep-ph]; P. Boucaud, J. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, *et al.*, JHEP **0806**, 099 (2008), arXiv:0803.2161 [hep-ph]; D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. **D78**, 065047 (2008), arXiv:0806.4348 [hep-th]; D. Binosi and J. Papavassiliou, Phys. Rept. **479**, 1 (2009), arXiv:0909.2536 [hep-ph]; P. Boucaud, J. Leroy, A. L. Yaouanc, J. Micheli, O. Pene, *et al.*, Few Body Syst. **53**, 387 (2012), arXiv:1109.1936 [hep-ph].
 - [20] C. S. Fischer, A. Maas, and J. M. Pawłowski, Annals Phys. **324**, 2408 (2009), arXiv:0810.1987 [hep-ph].
 - [21] F. Marhauser and J. M. Pawłowski, (2008), arXiv:0812.1144 [hep-ph]; J. Braun, L. M. Haas, F. Marhauser, and J. M. Pawłowski, Phys. Rev. Lett. **106**, 022002 (2011), arXiv:0908.0008 [hep-ph]; K.-I.

- Kondo, Phys. Rev. **D82**, 065024 (2010), arXiv:1005.0314 [hep-th].
- [22] L. Fister and J. M. Pawłowski, (2011), arXiv:1112.5429 [hep-ph]; (2011), arXiv:1112.5440 [hep-ph].
- [23] K. Fukushima, Phys. Lett. **B695**, 387 (2011), arXiv:1006.2596 [hep-ph].
- [24] J. M. Pawłowski, AIP Conf. Proc. **1343**, 75 (2011), arXiv:1012.5075 [hep-ph].
- [25] A. Cucchieri, T. Mendes, and A. R. Taurines, Phys. Rev. **D67**, 091502 (2003), arXiv:hep-lat/0302022 [hep-lat]; A. Cucchieri and T. Mendes, PoS **FACESQCD**, 007 (2010), arXiv:1105.0176 [hep-lat].
- [26] K.-I. Kondo, Phys. Rev. **D84**, 061702 (2011), arXiv:1103.3829 [hep-th].
- [27] D. Zwanziger, Phys. Rev. Lett. **94**, 182301 (2005), arXiv:hep-ph/0407103 [hep-ph].
- [28] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, *et al.*, Nucl. Phys. **B469**, 419 (1996), arXiv:hep-lat/9602007 [hep-lat]; O. Kaczmarek, F. Karsch, P. Petreczky, and F. Zantow, Phys. Lett. **B543**, 41 (2002), arXiv:hep-lat/0207002 [hep-lat].
- [29] Private communications with Jan Pawłowski.
- [30] S. Borsanyi, G. Endrodi, Z. Fodor, S. Katz, and K. Szabo, JHEP **1207**, 056 (2012), arXiv:1204.6184 [hep-lat].
- [31] C. S. Fischer, A. Maas, and J. A. Müller, Eur. Phys. J. **C68**, 165 (2010), arXiv:1003.1960 [hep-ph].
- [32] N. Ishii, H. Suganuma, and H. Matsufuru, Phys. Rev. **D66**, 014507 (2002), arXiv:hep-lat/0109011 [hep-lat]; Y. Hatta and K. Fukushima, Phys. Rev. **D69**, 097502 (2004), arXiv:hep-ph/0307068 [hep-ph].
- [33] J. Braun and A. Janot, Phys. Rev. **D84**, 114022 (2011), arXiv:1102.4841 [hep-ph]; J. Braun and T. K. Herbst, (2012), arXiv:1205.0779 [hep-ph].
- [34] T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994), arXiv:hep-ph/9401310 [hep-ph].
- [35] S. Borsanyi *et al.* (Wuppertal-Budapest Collaboration), JHEP **1009**, 073 (2010), arXiv:1005.3508 [hep-lat].
- [36] S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, *et al.*, JHEP **1011**, 077 (2010), arXiv:1007.2580 [hep-lat].
- [37] K. Kashiwa, Phys. Rev. **D83**, 117901 (2011), arXiv:1104.5167 [hep-ph].
- [38] R. Gatto and M. Ruggieri, (2012), arXiv:1207.3190 [hep-ph].
- [39] K. Kashiwa, T. Hell, and W. Weise, Phys. Rev. **D84**, 056010 (2011), arXiv:1106.5025 [hep-ph].